Deciding on a formula.

① Simple Interest?

Yes → $A = P(1+rt)$, $I = A - P = Prt$

No → ② Compounded Continuously?

Yes → $A = Pe^{rt}$, $I = A - P$

No → ③ Are there regular payments, deposits, or withdrawals?

No → $A = P\left(1+\frac{r}{m}\right)^{n}$, $I = A - P$

Yes → ④ Which comes first: regular payments or large sum of money?

large sum → $PV = PMT\left(\frac{1-(1+\frac{r}{m})^{-n}}{\frac{r}{m}}\right)$, $I = n \cdot PMT - PV$

payments → $FV = PMT\left(\frac{(1+\frac{r}{m})^{n}-1}{\frac{r}{m}}\right)$, $I = FV - n \cdot PMT$

Finite Math - Spring 2017

Review - 3/1/2017

Example 1. *If you invest* \$5,650 *in an account paying* 8.65% *compounded continuously, how much money will be in the account at the end of* 10 *years?*

$$A = Pe^{rt}$$

$$A = \$5,650e^{(0.0865)(10)}$$

$$= \$5,650e^{0.865}$$

$$= \boxed{\$13,418.78}$$

Example 2. *A company establishes a sinking fund for plant retooling in 6 years at an estimated cost of* \$850,000. *How much should be invested semiannually into an account paying* 8.76% *compounded semiannually?*

$\rightarrow m=2$

$$FV = PMT\left(\frac{\left(1+\frac{r}{m}\right)^{n}-1}{\frac{r}{m}}\right)$$

$$\$850{,}000 = PMT\left(\frac{\left(1+\frac{0.0876}{2}\right)^{12}-1}{\frac{0.0876}{2}}\right)$$

$$= PMT\left(15.35752008\right)$$

$$\Rightarrow \boxed{PMT = \$55{,}347.48}$$

no mention of compounding

Example 3. *If an investor wants to earn an annual interest rate of* 6.4% *on a* 26*-week T-bill with a maturity value of* \$5,000*, how much should the investor pay for the T-bill*

$$A = P(1+rt)$$

52 weeks per year

$$\$5{,}000 = P\left(1+(0.064)\left(\frac{26}{52}\right)\right)$$

$$= P(1.032)$$

$$\Rightarrow \boxed{P = \$4{,}844.96}$$

Example 4. *Which is the better investment and why:* 9% *compounded quarterly or* 9.25% *compounded annually?*

① ② APY

① $APY = \left(1 + \frac{0.09}{4}\right)^4 - 1 \approx 9.308\%$

② $APY = \left(1 + \frac{0.0925}{1}\right)^1 - 1 = 9.25\%$

① is better.

Example 5. *How long will it take* \$4,000 *to grow to* \$10,000 *if it is invested at* 6% *compounded monthly? 9% compounded monthly?*

① ②

$$A = P\left(1 + \frac{r}{m}\right)^{n}$$

① $\$10,000 = \$4,000\left(1 + \frac{0.06}{12}\right)^{n}$

$= \$4,000\,(1.005)^{n}$

$\Rightarrow 2.5 = (1.005)^{n}$

$\Rightarrow \ln 2.5 = \ln\left((1.005)^{n}\right) = n \ln(1.005)$

$\Rightarrow n = \frac{\ln 2.5}{\ln 1.005} \approx 183.7159$ months

184 months

② $\$10,000 = \$4,000\left(1 + \frac{0.09}{12}\right)^{n}$

$\Rightarrow 2.5 = (1.0075)^{n}$

$\Rightarrow \ln(2.5) = \ln\left((1.0075)^{n}\right) = n \ln(1.0075)$

$\Rightarrow n = 122.62987\ldots$

123 months

Example 6. *A couple has a* \$50,000, *10-year mortgage at* 9% *compounded monthly. What will their monthly payment be?*

$$PV = PMT\left(\frac{1-\left(1+\frac{r}{m}\right)^{-n}}{\frac{r}{m}}\right)$$

$$\$50{,}000 = PMT\left(\frac{1-\left(1+\frac{0.09}{12}\right)^{-120}}{\frac{0.09}{12}}\right)$$

$$= PMT\,(78.94169267)$$

$$\Rightarrow \boxed{PMT = \$633.38}$$

Example 7. *A person wants to establish an annuity for retirement purposes. He wants to make quarterly deposits for 20 years so that he can then make quarterly withdrawals of $5,000 for 10 years. The annuity earns 7.32% interest compounded quarterly.*

(a) *How much will have to be in the account at the time he retires?*

(b) *How much should be deposited each quarter for 20 years in order to accumulate the required amount?*

(c) *What is the total amount of interest earned during the 30-year period?*

(a) $PV = PMT\left(\frac{1-(1+\frac{r}{m})^{-n}}{\frac{r}{m}}\right)$

$PV = \$5,000\left(\frac{1-(1+\frac{0.0732}{4})^{-40}}{\frac{0.0732}{4}}\right) = \boxed{\$140,945.57}$

(b) $FV = PMT\left(\frac{(1+\frac{r}{m})^{n}-1}{\frac{r}{m}}\right)$

$\$140,945.57 = PMT\left(\frac{(1+\frac{0.0732}{4})^{80}-1}{\frac{0.0732}{4}}\right)$

$= PMT(178.4906925)$

$\Rightarrow \boxed{PMT = \$789.65}$

(c) $Int = 40(\$5,000) - 80(\$789.65) = \boxed{\$136,828}$

Example 8. *A couple wishes to have* \$40,000 *in* 6 *years for the down payment on a house. If the couple has* \$25,000 *to invest, what interest rate (a) compounded quarterly, (b) compounded monthly, (c) compounded continuously should the couple look for?*

(a), (b) $A = P\left(1+\frac{r}{m}\right)^{n}$ (c) $A = Pe^{rt}$

(a) $\$40,000 = \$25,000\left(1+\frac{r}{4}\right)^{24}$

$\Rightarrow 1.6 = \left(1+\frac{r}{4}\right)^{24}$

$\Rightarrow 1+\frac{r}{4} = \sqrt[24]{1.6} \approx 1.019776$

$\Rightarrow r \approx 0.07911 = 7.911\%$

(b) $\$40,000 = \$25,000\left(1+\frac{r}{12}\right)^{72}$

$\Rightarrow 1.6 = \left(1+\frac{r}{12}\right)^{72}$

$\Rightarrow \sqrt[72]{1.6} = 1+\frac{r}{12}$

$\Rightarrow r \approx 0.07859 = 7.859\%$

(c) $\$40,000 = \$25,000\, e^{r(6)} \Rightarrow 1.6 = e^{6r}$

$\Rightarrow \ln 1.6 = \ln(e^{6r}) = 6r \ln e = 6r$

$\Rightarrow r \approx 0.07833 = 7.833\%$